

## Estimation of Factor Demand and Output Supply of Sorghum

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### Abstract

The output supply and factor demand are closely interlinked to each other. Therefore, any change in factor and product prices affect the input demand and output supply simultaneously. The present study used cross sectional cum time series data of Vidarbha region of Maharashtra state for sorghum crop for the ten years from 1999-2000 to 2008-09. The analysis of factor demand equation showed that the demand elasticities with respect to own prices had the expected negative sign indicating that the results were in accordance with the theory of demand. The absolute value of own price elasticity of human labour, bullock labour and seed were greater than unity indicating there by an elastic response of input utilization to their own price. One per cent increase in own price, holding other prices constant, will reduce human labour employment at 1.26%, bullock labour demand 1.134% and seed demand 1.08% in sorghum crop. Joint estimation of Cobb-Douglas profit function and factor demand equation by using Zellner's method shows that the profit function was decreased in prices of bullock labour and Fertilizer.

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**Keywords:** Input demand, sorghum, Vidarbha, profit function.

### Introduction

The price of agricultural commodities and agricultural raw materials are the key factors in the price structure of an agrarian economy. Lower prices of output act as a deterrent in the utilization of inputs while higher output prices may bring about additional utilization of resources

for maximization of profit whereas the rising input prices discourage input use and vice versa. The decline in output supply raises food prices. The rapid increase in population and the increase in money income as a result of economic development create a strong pressure on demand which also leads to increase in food grain prices. These cause hardship to the consumers. This can be corrected only by a large and adequate supply of agricultural output and a greater attention is, therefore, required to be focused for matching the demand for foodgrains and agricultural commodities with the supply thereof. The rise in foodgrains prices should be sufficiently high not only to counteract the rising cost of inputs but also to leave a rate of profit conducive for investment in agriculture and thereby accelerating supply of agricultural output. In this context, one needs detailed knowledge about the net effect of price and non-price factors like factor and product prices, technology, irrigation, capital use, acreage etc. so that required adjustment needed in price and non-price factors could be worked out to attain the specific goals of prices, production and crop income.

Sorghum is one of the important cereal crop in India. Sorghum ranks fifth, among the world cereal food crops (After Rice, Wheat, Maize, and Barley). The main states of India where the sorghum is cultivated are Maharashtra, Karnataka, Madhya Pradesh, Andhra Pradesh and Rajasthan. In India during 2008-09, the area under sorghum was 7.70 million hectares with production of 7.30 million tonnes and productivity was 952 kg ha<sup>-1</sup> (Anonymous a 2009) while in Maharashtra the area under sorghum was 4.17 million hectare with production of 3.67 million tonnes and productivity was 877 kg ha<sup>-1</sup> and the area in Vidarbha under sorghum cultivation was 2.88 lakh hectares with production of 3.77 lakh tonnes and productivity was 1308 kg ha<sup>-1</sup>.

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## Methodology

The Agricultural Prices and Costs (APC) scheme under the guidance of government of Maharashtra provides valuable data about Agriculture in Maharashtra. The present study used cross sectional cum time series data of Vidarbha region for the sorghum crop for the ten years i.e. from 1999-2000 to 2008-09. Every year 100 farmers were selected for the present study. The scheme is involved in the collection of representative data on input use and yield and there upon estimation of cost of cultivation of principle crops grown in the region. Data is collected every year and for all the enterprises. Although, the sample for particular year are selected with respect to specified principle crop. The data were collected for all the crops grown on the sample holdings.

### Index of Input Prices

The input price indices are composite indices of prices of individual items of inputs. The indices were constructed using the cost of cultivation data for the period of last ten years with average of first triennium ending as the base year. First, the price indices of inputs of seed, labour, bullock labour, fertilizer, farm yard manure, capital, pesticide and depreciation on implements were constructed.

The composite indices of input prices for Sorghum crop were constructed as

$$\text{Index of Input Price} = \sum_{i=1}^9 S_i \left( \frac{P_{it}}{P_{i0}} \right)$$

Where,  $S_i$  = average share of  $i$ th input in total input cost.  $P_{it}/P_{i0}$  is the price index of  $i^{\text{th}}$  input in the  $t^{\text{th}}$  year using average of first triennium as the base year,  $i=1$  stands for human wage index,  $i=2$  bullock wage index,  $i=3$  fertilizer price index,  $i=4$  FYM price index,  $i=5$  seed price index  $i=6$  interest rate index,  $i=7$  pesticide expenditure index,  $i=8$  depreciation charges index, and  $i=9$  rental value of land index.

### Mathematical Model

Profit Function Analysis: The theory of profit function was developed to help in overcoming the problem of simultaneous equation bias, if present. Another distinct

advantage of this approach over production function is that with the help of duality theorem (Shephard 1953), the variable factor demand function and supply function of products can be derived directly from the estimated profit function. Econometric application of this production theory based on duality between production function and variable profit function is a breakthrough in the theory of production. Shephard's Lemma (1953) applies equally to profit functions, which states that the partial derivative of profit function with respect to output and input prices give the supply and demand function, respectively.

Let the Cobb-Douglas production function with usual neo-classical properties be written as

$$Q = A N^{\alpha_1} B^{\alpha_2} X^{\alpha_3} F^{\alpha_4} S^{\alpha_5} K^{\beta_1} L^{\beta_2} U$$

Where,  $Q$  is output of crop;  $N$  as human labour,  $B$  as bullock labour,  $X$  as chemical plant nutrients,  $F$  as farm yard manure and  $S$  as seed are the variable inputs; and  $K$  as capital and  $L$  as land are fixed inputs; and  $U$  is error term.

When working with profit function one has to choose functional forms which are homogenous of degree one in all prices, whereas this is not necessary for normalized profit function. The profit function formulation suggested by Lau and Yotopoulos (1972) enables us to derive factor demand as a function of normalized input rates and the quantities of fixed inputs.

Invoking the theory of profit function, the normalized profit function for the above production function can be written as below.

$$\frac{\pi}{P} = A^* \left( \frac{w}{P} \right)^{\alpha_1^*} \left( \frac{b}{P} \right)^{\alpha_2^*} \left( \frac{r}{P} \right)^{\alpha_3^*} \left( \frac{m}{P} \right)^{\alpha_4^*} \left( \frac{S}{P} \right)^{\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*} U$$

OR

$$\pi^* = A^* w^{*\alpha_1^*} b^{*\alpha_2^*} r^{*\alpha_3^*} m^{*\alpha_4^*} S^{*\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*} U$$

Where  $\pi^* = \pi/P$  = normalized profit or output price (UOP) profit,  $w^*$  is the normalized wage rate,  $b^*$  is the normalized bullock labour price,  $r^*$  is the normalized fertilizer price,  $m^*$  is the normalized farm yard manure

price and  $s^*$  is the normalized seed price.

From the estimated parameters of normalized profit function, the production elasticities of inputs and intercept were derived.

*Factor Demand Function*

Shepherd's Lemma (1953) asserts that the first order negative derivative of the normalized profit equation with respect to normalized wage rate, bullock labour price, fertilizer price, farm yard manure price and seed price respectively, gives the derived factor demand function.

The factor demand equation on case of Cobb-Douglas type normalized profit function was given as

i) Human labour demand equation

$$-\frac{\partial \pi^*}{\partial w^*} = -\alpha_1^* \left(\frac{\pi^*}{w^*}\right) = N \text{ or } \frac{w^*N}{\pi^*} = -\alpha_1^* \dots\dots(a)$$

ii) Bullock labour demand equation

$$-\frac{\partial \pi^*}{\partial b^*} = -\alpha_2^* \left(\frac{\pi^*}{b^*}\right) = B \text{ or } \frac{b^*B}{\pi^*} = -\alpha_2^* \dots\dots(b)$$

iii) Fertilizer demand equation

$$-\frac{\partial \pi^*}{\partial r^*} = -\alpha_3^* \left(\frac{\pi^*}{r^*}\right) = X \text{ or } \frac{r^*X}{\pi^*} = -\alpha_3^* \dots\dots(c)$$

iv) Farm-yard manure demand equation

$$-\frac{\partial \pi^*}{\partial f^*} = -\alpha_4^* \left(\frac{\pi^*}{f^*}\right) = F \text{ or } \frac{f^*F}{\pi^*} = -\alpha_4^* \dots\dots(d)$$

v) Seed demand equation

$$-\frac{\partial \pi^*}{\partial s^*} = -\alpha_5^* \left(\frac{\pi^*}{s^*}\right) = S \text{ or } \frac{s^*S}{\pi^*} = -\alpha_5^* \dots\dots(e)$$

Substituting  $\pi^*$  from identity (1) into (a) to (e), the demand equation can be written as:

i) Human labour demand equation

$$N = -\alpha_1^* A^* (w^*)^{\alpha_1^*-1} b^{*\alpha_2^*} r^{*\alpha_3^*} m^{*\alpha_4^*} s^{*\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*}$$

ii) Bullock labour demand equation

$$B = -\alpha_2^* A^* w^{*\alpha_1^*} (b^*)^{\alpha_2^*-1} r^{*\alpha_3^*} m^{*\alpha_4^*} s^{*\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*}$$

iii) Fertilizer demand equation

$$X = -\alpha_3^* A^* w^{*\alpha_1^*} b^{*\alpha_2^*} (r^*)^{\alpha_3^*-1} m^{*\alpha_4^*} s^{*\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*}$$

iv) Farm-yard manure demand equation

$$F = -\alpha_4^* A^* w^{*\alpha_1^*} b^{*\alpha_2^*} r^{*\alpha_3^*} (m^*)^{\alpha_4^*-1} s^{*\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*}$$

v) Seed demand equation

$$S = -\alpha_5^* A^* w^{*\alpha_1^*} b^{*\alpha_2^*} r^{*\alpha_3^*} m^{*\alpha_4^*} (s^*)^{\alpha_5^*-1} K^{\beta_1^*} L^{\beta_2^*}$$

*Output supply Function*

Shepherd's Lemma (1953) asserts that first order derivative of profit function with respect to output price gives output supply function.

$$\frac{\partial \pi}{\partial P} = \theta \left(\frac{\pi}{P}\right) = Q \text{ or } \frac{P \times Q}{\pi} = \theta$$

The output supply function in the form of Cobb-Douglas production function was written as

$$Q = A \theta P^{\theta-1} w^{*\alpha_1^*} b^{*\alpha_2^*} r^{*\alpha_3^*} m^{*\alpha_4^*} s^{*\alpha_5^*} K^{\beta_1^*} L^{\beta_2^*}$$

The above equation was giving the output supply with respect to output prices, wage rate, bullock labour price, fertilizer price, farm yard manure price, seed price and price of capital input.

*Joint Estimation of Cobb-Douglas profit functions and factor demand.*

The normalized profit function and factor demand functions for human labour, bullock labour, fertilizer farmyard manure and seed were jointly estimated using Zellner's method (1962) for estimating 'Seemingly Unrelated Regression Equation (SURE)' by imposing the restriction that  $\alpha_1^*$ ,  $\alpha_2^*$ ,  $\alpha_3^*$ ,  $\alpha_4^*$  and  $\alpha_5^*$  are equal in both the normalized profit function and relevant factor demand equations.

By using SURE method the coefficient were estimated as

$$\hat{\alpha}_{SURE} = (X' V^{-1} X)^{-1} X' V^{-1} Y$$

Where, X is independent variable and Y is dependent

variable

$$V = \sum \oplus I_N$$

Where,  $\sum$  representing the covariance of residual between the equations,  $\oplus$  is the Kronecker product and  $I_N$  is the identity matrix of number of observations.

### Results and Discussion

#### Changes in input and output prices

Transformation of agriculture from subsistence to profitable farm business is a techno-organizational process, the success of which largely depends on the relative prices of various inputs and outputs. Therefore, it would be interest to examine the changes in prices of inputs and outputs.

#### Compound growth rates of input and output prices

The rate of growth of average input prices and output prices for sorghum crop are presented in Table 1. It shows that the compound growth rate of input prices for sorghum were highest for Farm Yard Manure (12.31% per annum) followed by prices of bullock labour (5.60% per annum). The per cent growth rates in prices of human wage rate and seed were observed to be 5.15% and 4.06% per annum respectively. The output prices increased at an annual compound rate of 8.06% per annum for sorghum during the period under study.

**Table 1.** Compound growth rates of input and output prices of Sorghum

Items	Sorghum
1) Input Prices	
i) Wage rate	5.15***
ii) Bullock labour price	5.60***
iii) FYM price	12.31*
iv) Fertilizer price	-0.43
v) Seed price	4.06***
2) Output Price	7.06***

(\*\*\*, \*\*, \* denotes significant at 1%, 5%, and 10% level of significance)

#### Parity between prices received for products and prices paid for inputs

Parity prices for farm products are those prices which would give the same purchasing power to the producer as prevailed in the base year. In order to examine the parity between the prices received for output and prices paid for agricultural inputs, parity indices were computed by deflating output price indices by the input price indices.

#### Parity between output price index and input price index for sorghum

Table 2 presents input-output price indices for sorghum crop. It is evident from the table that between 1999-2000 to 2008-09, the input price index for sorghum increased by 28 per cent, while the increase in output price was 88 per cent. Further, the output-input price parity were decreased during year 2001-02 and 2003-04, increased in the subsequent years, indicating thereby up to the year 2001-02 and in 2003-04, the output price were lower than input price and term of trade was unfavourable for sorghum growers. However, the term of trade was favourable for the sorghum growers afterward.

#### Input Demand Function

**Table 2.** Parity between output price index and input price index for sorghum (Base year- Average of Triennium Ending – 1999-2000 to 2001-02)

Years	Input price Index	Output price Index	Parity Index
1999-00	111.77	119.19	106.64
2000-01	92.98	108.37	116.55
2001-02	97.07	72.44	74.63
2002-03	100.63	104.54	103.88
2003-04	103.89	102.98	99.12
2004-05	110.90	116.99	105.50
2005-06	118.09	124.55	105.47
2006-07	115.26	150.24	130.35
2007-08	130.44	162.18	124.33
2008-09	128.72	188.64	146.56

A system of factor demand equations were derived from the estimated normalized profit function. The results of human labour, bullock labour, fertilizer, farm yard manure and seed demand equation for sorghum are presented below and the degree of responsiveness of input and output price movements on the use of inputs are discussed. This information is of crucial importance in the formulation of effective price policies for crops to reach specified production goals.

#### *Input Demand Function for sorghum*

Table 3 revealed that demand elasticities with respect to own price had anticipated negative signs indicating that the results were in accordance with the theory of demand. The absolute value of own price elasticity of human labour, bullock labour and seed were greater than unity indicating there by an elastic response of input utilization to their own price.

One per cent increase in own price, holding other prices constant, will reduce human labour employment at 1.260%, bullock labour demand 1.134% and seed demand 1.080% in sorghum crop.

A negative sign of cross price elasticity with respect to the price of other variable inputs shows that the pair is complement and a positive sign is an indicator of substitutive relationship. However, the positive sign of cross price elasticity with respect to quantities of fixed inputs indicates complementarity and negative sign indicates substitutive relationship.

The output supply equations for sorghum in Vidarbha were derived from the estimated profit function. The output supply equation given in Table 8 gives the estimates of the responses of own output price, variable prices and fixed factors on output supply of selected crops.

The table shows that for the sorghum, variable inputs responded positively to the output price except bullock labour price. Reddy and Chengappa (1997) observed that the variable inputs responded positively to the output price except human labour and fertilizer price. The elasticity with respect to bullock labour price for sorghum was -0.02 resulted that a 1% increase in bullock price was associated with about 0.02% decline in crop output. However, among the fixed factor capital was found to be effective in increasing the supply of sorghum. Capital input had positive impact (0.458) on the supply of sorghum.

#### *Joint estimation of the Normalized profit functions and factor share for variable inputs*

Lau and Yotopoulos (1972) pointed out that due to the presence of common parameters in profit and factor demand equation; they should be estimated jointly imposing the restriction that common parameters in both equations are equal. The five equations - UOP profit function, human labour, bullock labour, fertilizer, farm yard manure and seed demand functions were estimated jointly using Zellner's method (1962) for estimating

**Table 3.** Input demand and Output Supply function for sorghum

Variables	Human Labour	Bullock Labour	Fertilizer	F.Y.M.	Seed	Output Supply
Intercept	0.561	0.292	-1.252	-0.381	0.042	-0.008
Output price	1.013	0.993	0.246	0.641	1.010	-1.026
Wage rate	-1.260	0.018	-0.347	-0.360	0.061	0.215
Bullock labour price	0.128	-1.134	-0.486	-0.149	-0.018	-0.020
Fertilizer price	0.020	0.020	0.292	-0.078	-0.002	0.014
F.Y.M. price	0.038	0.038	-0.048	-0.154	-0.001	0.054
Seed Price	0.064	0.208	-0.058	-0.109	-1.080	0.039
Capital	0.143	-0.024	0.384	0.202	0.114	0.458
Land	-0.146	-0.120	0.018	0.006	-0.083	-0.042

‘Seemingly Unrelated Regression Equation (SURE)’ by imposing appropriate restrictions.

The table shows that the profit function for sorghum was decreased in prices of bullock labour and fertilizer. Among the variable factors, normalized bullock labour price in general had the highest negative impact on variable profit followed by fertilizer price.

### Conclusion

From this study it is concluded that, the absolute value

of own price elasticity of human labour, bullock labour and seed were greater than unity indicating there by an elastic response of input utilization to their own price. One per cent increase in own price, holding other prices constant, will reduce human labour employment at 1.260%, bullock labour demand 1.134% and seed demand 1.080% in sorghum crop. Joint estimation of Cobb-Douglas profit function and factor demand equation by using Zellner’s method shows that the profit function was decreased in prices of bullock labour and Fertilizer.

**Table 4.** Joint estimation of the Normalized profit function and factor share for variable inputs for Sorghum

Variable	Parameters	SURE estimated Values
Normalized profit (dependent variable)	$\ln \pi/p$	
Wage rate	$\ln w/p$	0.183
Bullock labour price	$\ln b/p$	-0.118
Fertilizer price	$\ln r/p$	-0.009
Farm yard manure price	$\ln m/p$	0.052
Seed	$\ln s/p$	0.197
Capital input	$\ln K$	0.439
Land	$\ln L$	-0.056

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